



Rethinking Mathematics Instruction for Students with Significant Cognitive Disabilities



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Center for Literacy & Disability Studies

- Mission is to promote literacy learning and use for individuals of all ages with disabilities
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- <http://www.med.unc.edu/ahs/clds/>



Creating and Identifying Patterns (Introductions)

- Divide into small groups of four.
- Introduce yourself in the small groups and discuss a possible pattern that exists in your group. Examples of possible patterns (brown hair, blonde hair, brown hair, blonde hair) or (glasses, no glasses, glasses, no glasses, etc).



Introductions

- Pair with another group and stand so that the pattern the group has selected is visible to the other group.
- The other group will make conjectures about what the pattern could be. Write down the words used to describe the pattern. The small group will confirm and/or explain their pattern. Then reverse roles. Return to your seats.
- Who is in the audience? Raise your hand if...



Goal of Activity

To engage you in
Mathematical Thinking



Objectives

Participants will

- Discuss the use of core vocabulary and number lines to enhance student communication skills in the area of mathematics.
- Discuss big ideas associated with representing quantities.
- Use physical models to explore number relationships.



Objectives

- Apply knowledge in lesson plans.
- Discuss what it means to think algebraically.
- Describe fractions as the division of a whole into equal parts.
- Discuss composing and decomposing.



Instructional Approach

For Students with
Significant Cognitive Disabilities



Instruction for SCD

Typical Instruction

- Rote Counting
- One-to-One Matching
- Procedural Understanding

New Approach

- Conceptual & Procedural Understanding
- Context and Purpose
- Repetition with Variety
- Teacher/Peer Modeling
- Explicit Instruction



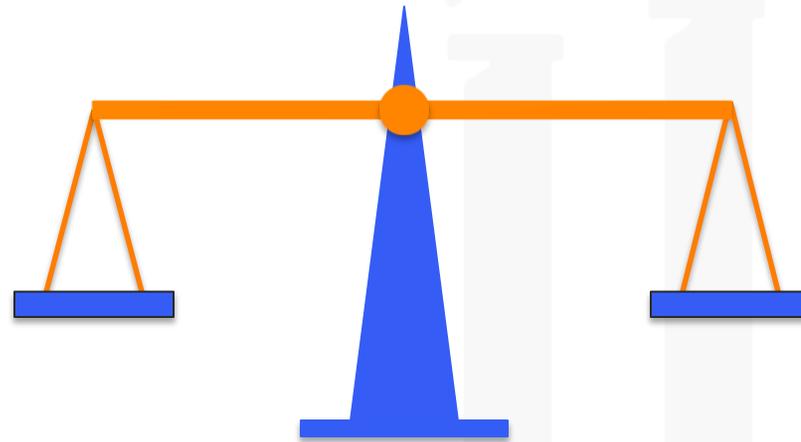
Procedural Knowledge Conceptual Knowledge

- Action sequences for solving problems
- Facts, skills, procedures, algorithms, methods
- Learning that involves memorization
- Knowing how something happens

- Ideas, relationships, connections
- To know why something happens in a particular way



Creating a Balance

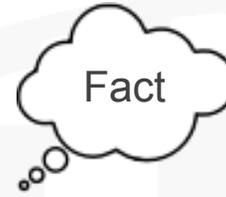
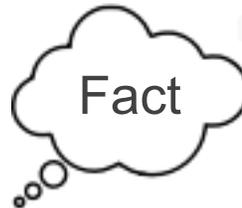


"...focus needs to be on relationships between conceptual and procedural knowledge in math"

Star (2002)



Context and Purpose



Learning facts without a context means they don't have anything to connect to. The just eventually float away.



- **Context**
 - Meaning of the mathematical concept is partly or wholly determined by the situation in which it is embedded.
 - Personally relevant
- **Purpose:**
 - Clearly state what the student is intended to think and do.



Repetition with Variety

- We know from both research and experience that students with significant cognitive disabilities benefit from repetition, but that repetition should offer as much variety as possible.
- When teachers instruct using *repetition with variety*, the student receives multiple opportunities to work on the target goal in different ways.



Teacher/ Peer Modeling

- Describe and model the math skill/concept.
- Describe feature of the math concept or steps in performing math skill.
- Describe/model using multi-sensory techniques.
- Engage students in learning through
 - » demonstrating enthusiasm
 - » maintaining a lively pace
 - » periodically questioning students
 - » checking for student understanding



Explicit Instruction

- Focus instruction on critical content.
- Review prior skills and knowledge before beginning instruction.
- Begin lessons with a clear statement of purpose.
- Provide guided and supported practice.
- Require frequent responses from the student.



- Use clear and concise language.
- Monitor student performance closely.
- Provide immediate affirmative and corrective feedback.
- Help students organize knowledge.
- Provide repetition with variety.



The Language of Mathematics

Look back at the words you wrote down to describe your group patterns. Share some of the descriptors.



Challenges to Participation in Math Discussions

- Language constraints
- Complex communication needs
- Feeling there is nothing to contribute
- Lack of success



Goals in Providing Support

- Provide ways to become involved in discussions
- Point out the value of their contributions
- Attempt to get them to expand on those contributions so that they can expand their own thinking and understanding



**Every student *must* have a
means of expressive
communication.**



AAC systems should include vocabulary that is used frequently and across settings.



Core Vocabulary

A relatively small set of highly useful words that apply across contexts.

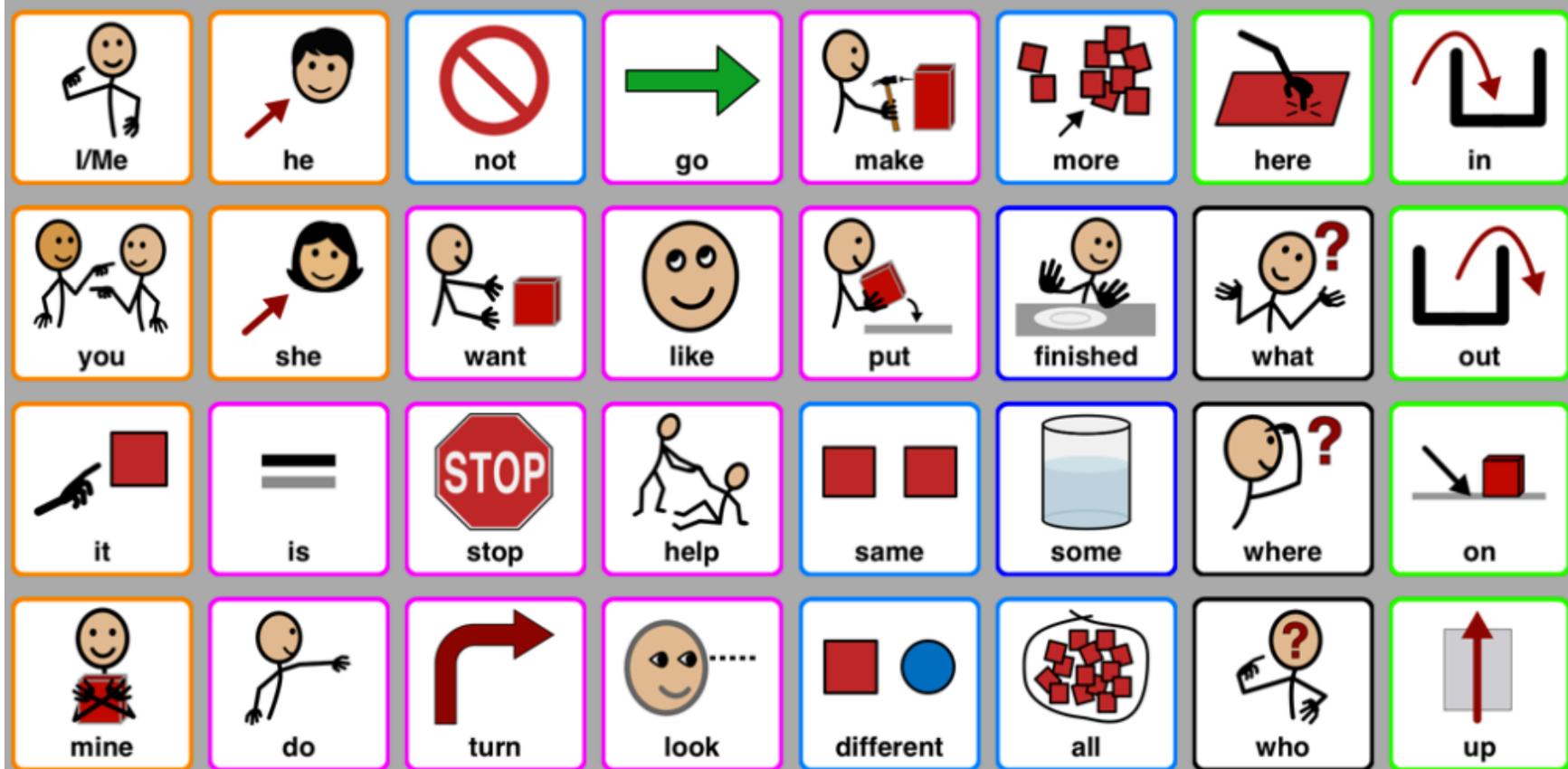


85% of spoken language comprised of 250 – 350 words

If we choose the correct vocabulary, students who use AAC can use this relatively small set of words to communicate many things across a variety of contexts.



Core Vocabulary Example

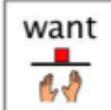
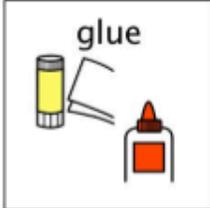
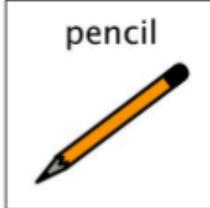
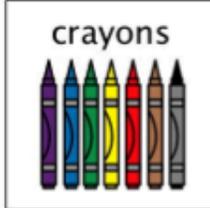
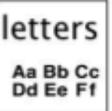
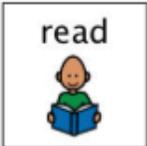
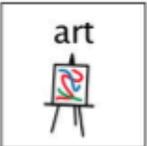
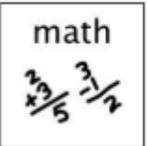
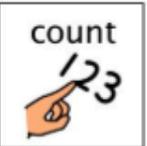
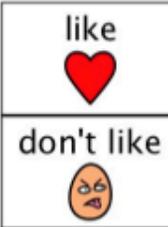
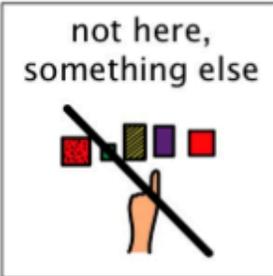


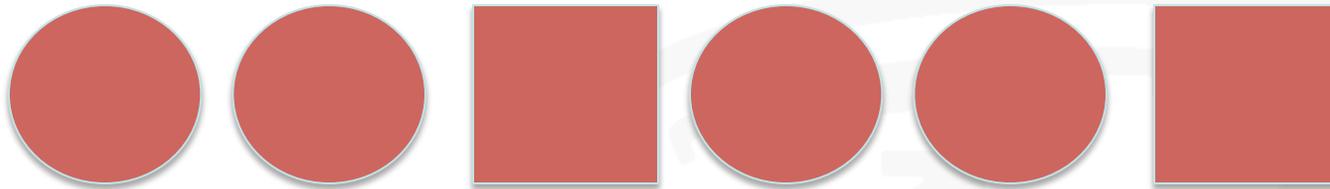
Contains pronouns, verbs, descriptors and question words rather than nouns which are often specific to a particular situation.



Vs. Activity Specific Words

Table Activities

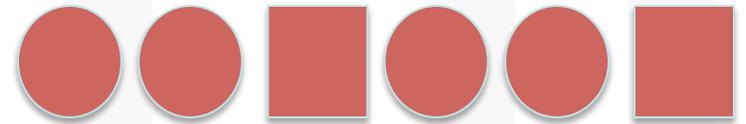


help	it
more	different

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---



I	like	not	want
help	it	more	different
who	she	you	he
where	up	on	in
me	make	get	look
what	need	are	is
some	put	all	this
don't	that	go	do
when	finished	can	here
open	turn	stop	over



0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---



Strategies to Support Participation in Discussions

Video



Strategy #1

Create a classroom climate that encourages thinking and contributing.



Strategy #2

Use probing questions to encourage thinking and expansion of ideas.



Strategy #3

Expand student responses.



Strategy #4

Offer response options.



Strategy #5

Presume competence and respond naturally.



Be Sure You Know

- What concepts you are teaching?
- How does the concept connect to other concepts the students may already know?
- What visuals or words will you use to communicate the content and language objectives?



- What context might you use that is meaningful to the student?
- Using the students core vocabulary and number line, what words will you model to communicate the concept?



Forms of Number and Ten Frames

What are the the strategies you typically use when you begin to teach students about quantity and numbers?



Forms of Number

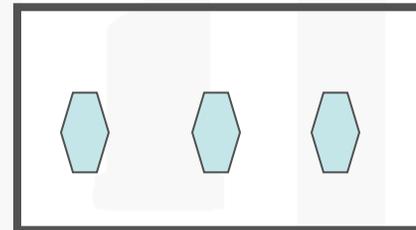
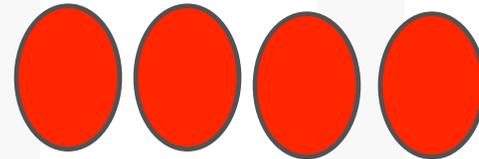
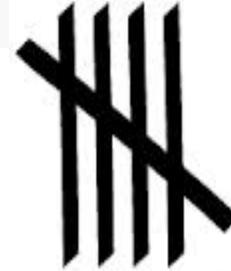
- Concrete





Forms of Number

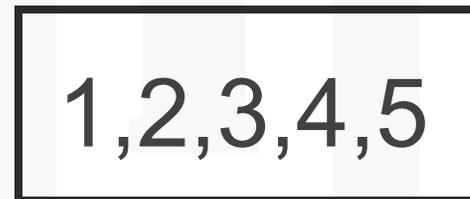
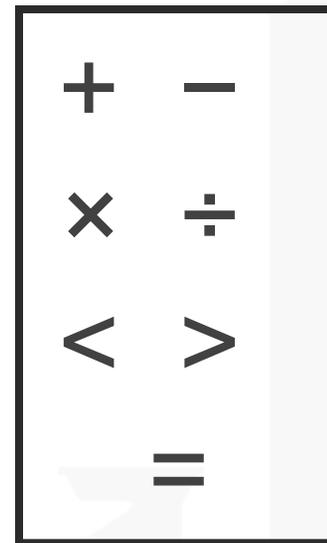
- Pictorial





Forms of Number

- Symbols and Numeral





Activity 1

Forms of Number Handout

As a small group fill in the blanks and discuss how you use each of these forms with your students.



Big Ideas

- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- There is a number word and a matching symbol that tell exactly how many items are in a group.
- Count to answer “how many?”



Big Ideas

- When counting, the last number tells the total number of items.
- The number of objects is the same regardless of their arrangement or the order in which they were counted.
- Quantities can be represented using objects, pictures, words, and symbols and numerals



Teaching Counting

- Pair numbers with meaningful objects.
- Count anything and everything.
- Provide experience with number lines.
- Read literature that involves numbers.
- Use five frames and ten frames to compare numbers.
- Collect and chart data.



Great Teaching Idea



Counting Hands can be used to represent numbers 0-10 paired with number cubes, dot cards, numerals or other concrete materials.

Retrieved From:

<http://www.jdaniel4smom.com/2013/10/counting-hands-math-activity.html>



Activity 2

Make Sets of More/Less/Same
Handout

Read the lesson. Are there adaptations that need to be made to the lesson for your specific group of students? (physical or visual impairments, language issues, etc.)

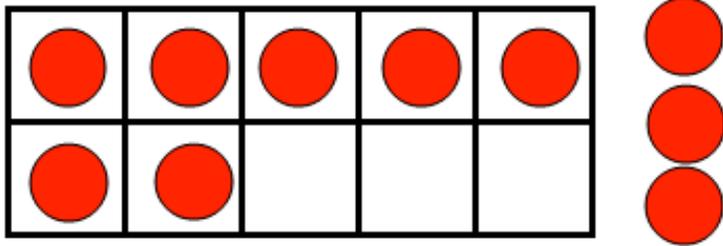


Ten Frames

A tool with many uses!
Developing number sense within the
context of ten.



Ten-Frames



Paper Ten-Frame



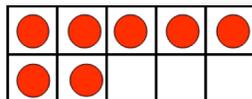
Boxed Ten-Frame



Life Size Ten-Frame

Rules of Working with Five and Ten-Frames

- One counter is permitted in each section of the frame.
- Fill the sections from left to right, top to bottom

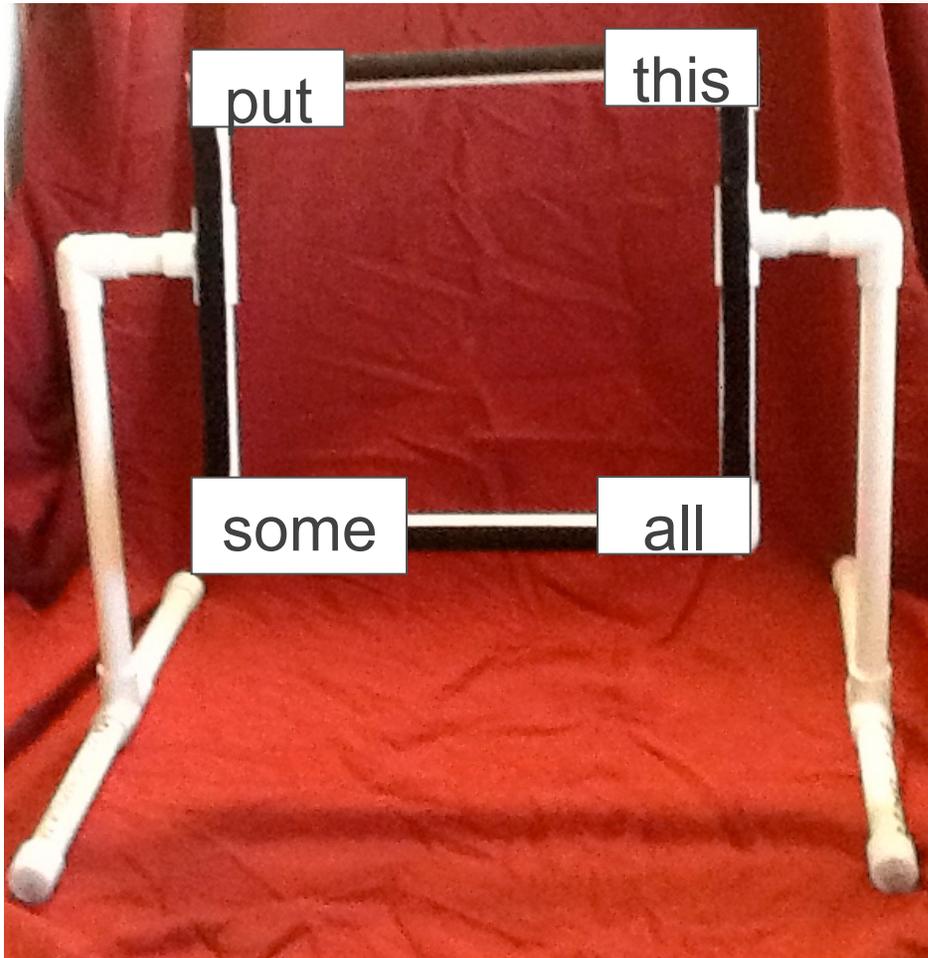


- or fill in pairs bottom to top in a vertical format





Expressive Communication



Eye Gaze Frame

●	●	●		

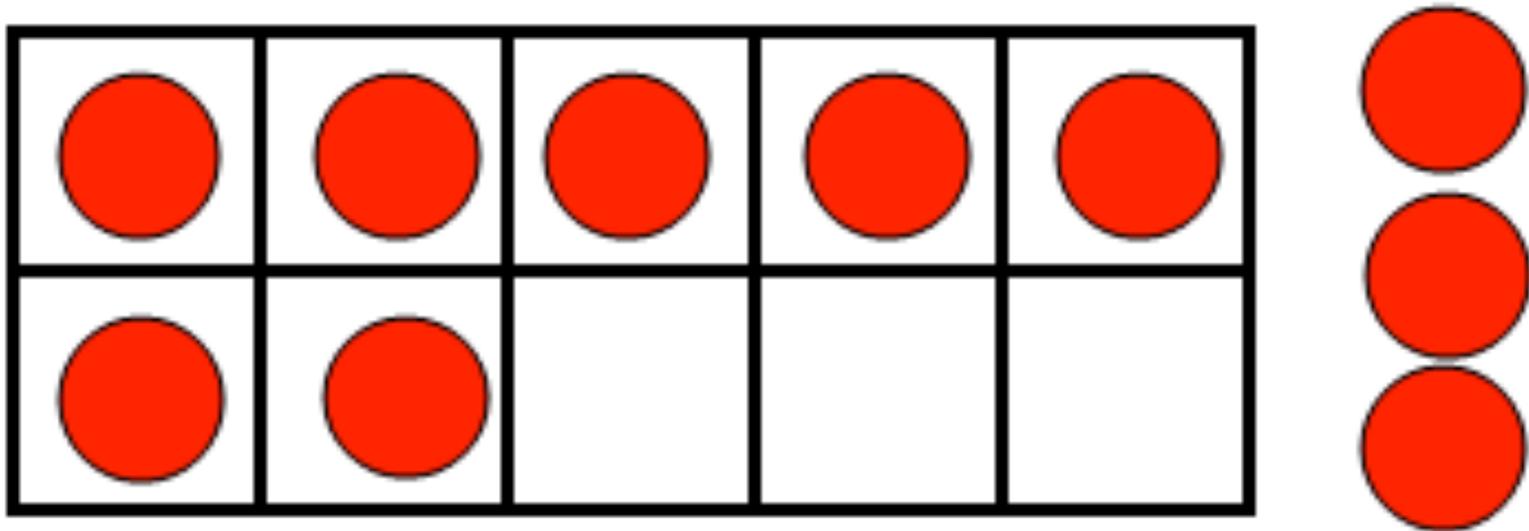
Ten-Frame

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

Number line



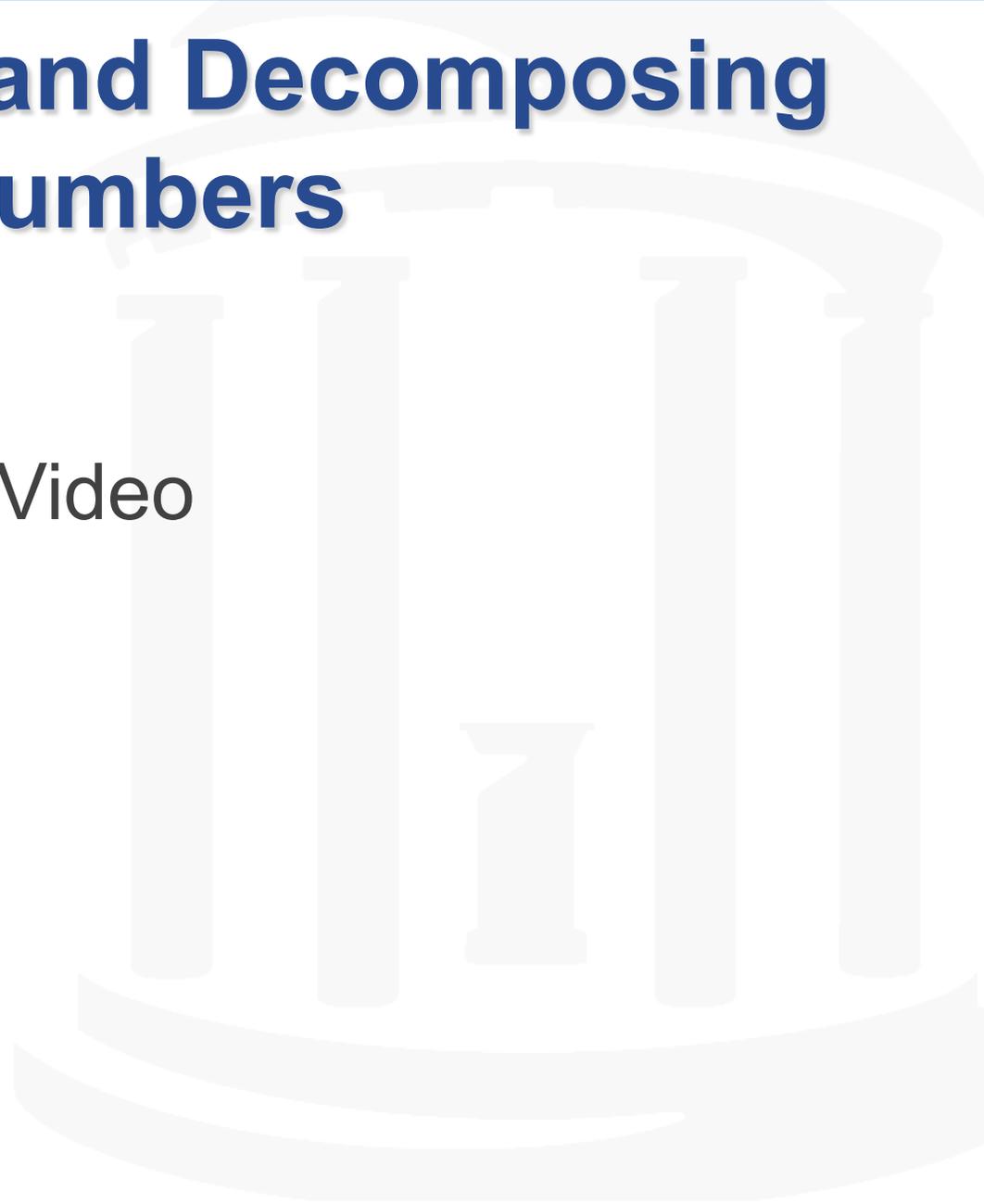
Composing and Decomposing Numbers





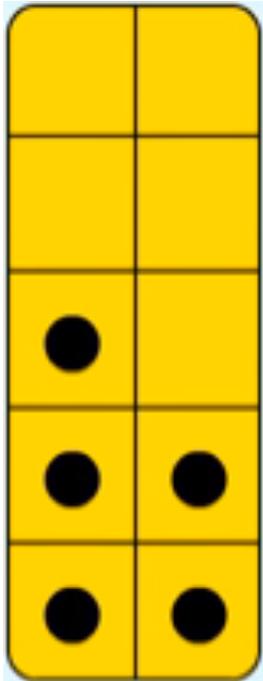
Composing and Decomposing Numbers

Video

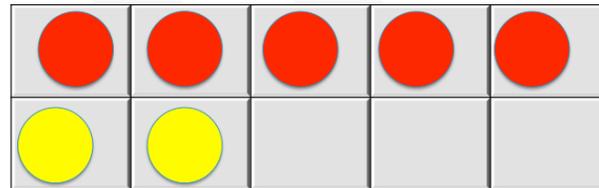




How Many?



I have 5, how many more do I need to make 10?

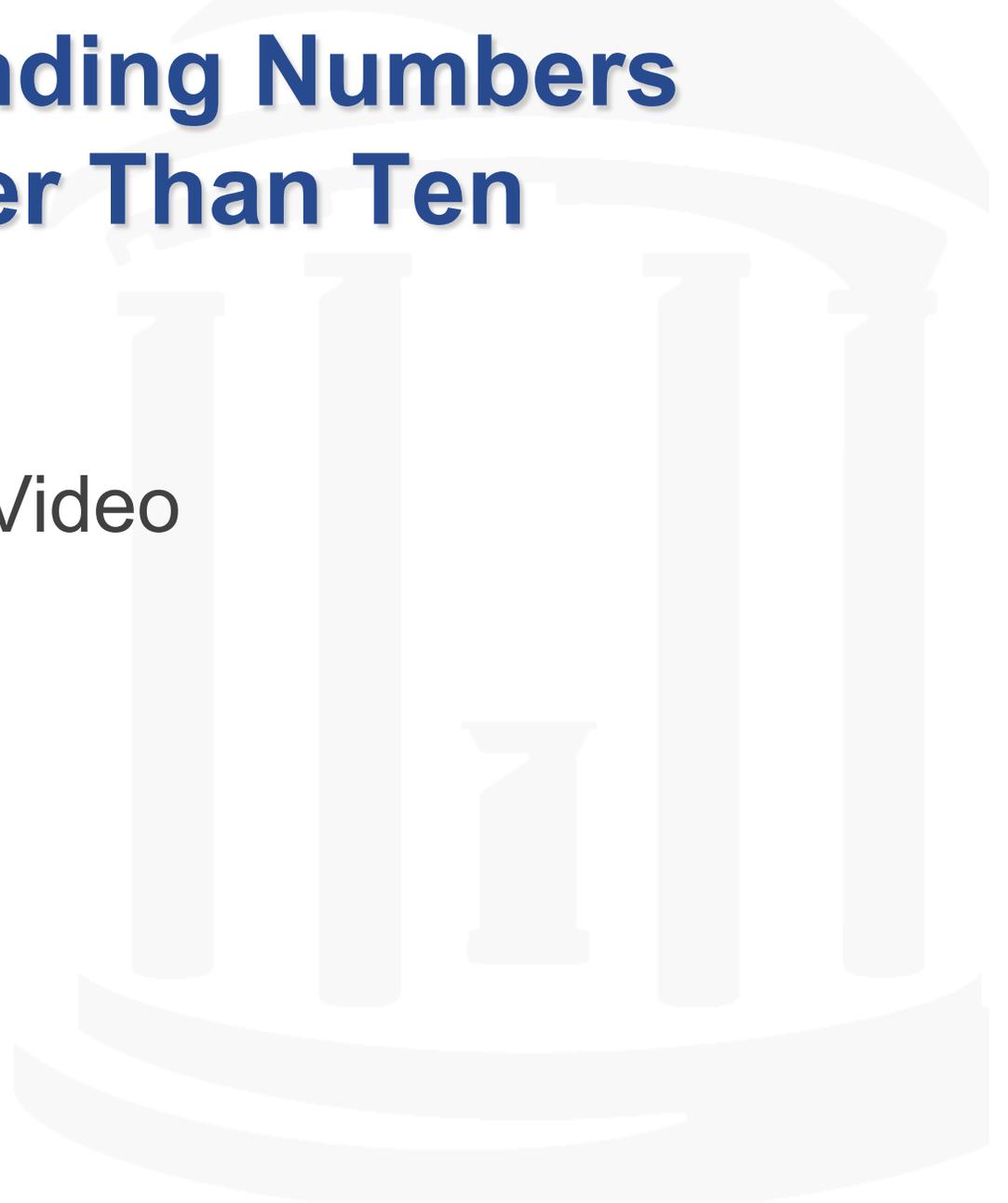


5 and 2 equals 7. How many do I need to make 10?



Understanding Numbers Greater Than Ten

Video





Break



Anchor, Think and Do, Apply

Lesson Planning vs. Schedules



Anchor

Think and Do

Apply



Anchor

- Activate prior knowledge
- Teach required new knowledge
- State purpose for lesson
 - » Clearly state what the student is intended to think and do.



Think and Do

- Students **think** about the task and then **do** one of the following:
 - » Solve the problem
 - » Build the model
 - » Find the matching shapes
 - » Put things in order
 - » Interpret the data, etc.



Apply

- Students describe or show what was done
- Teachers give informative feedback
- Students and teachers work together to restate how students **think** and **do** when faced with similar problems in the future.



Activity 3

Find the lesson plan titled Perimeter and the blank Think and Do, Apply lesson template. The Perimeter lesson is an example lesson that can help guide you as you write your own lesson.

1. In your small groups, write one lesson based on the forms of number and ten-frame information discussed previously.
2. We will discuss a couple of them as a group when you finish
3. You have 20 minutes to write your plan.



Lunch



Calculating Accurately with Addition



Combining Action

- Addition *combines* any two or more numbers
 - $4+4$
 - $4+5+8$
 - $127 + 456$



Calculating Accurately

- Number relationships

$$3 + 5$$

$$6 + 5 =$$

$$5 + 5 = 10$$

6 + 5 is one more

$$6 + 5 = 11$$

$$3 + 3 = 6$$

$$3 + 9 =$$

$$9 + 1 = 10$$

Plus 2 more

$$10 + 2 = 12$$



Calculating Accurately

- Provide experience with all problem structures

$$8 + 7 = \underline{\quad}$$

$$8 + \underline{\quad} = 15$$

$$\underline{\quad} + 7 = 15$$



Calculating Accurately

- Teach properties

Commutative Property

$$2 + 3 = 3 + 2$$

Associative Property

$$2 + (3 + 4) = (2 + 3) + 4$$

Identity Property

$$2 + 0 = 2$$



Activity 4

Making Numbers

Read the lesson plan.

Respond to the questions keeping in mind that active engagement is a key to learning.



Fact Mastery

Strategy	Representative use to solve
Counting all	"1, 2 . . . 1, 2, 3, 4 . . . 1, 2, 3, 4, 5, 6"
Short-cut sum	"1, 2, 3, 4, 5, 6"
Finger display	Displays 2 fingers, then 4 fingers, says "6"

Garnett, K. (1992). Developing fluency with basic number facts: Intervention for students with learning disabilities. Retrieved from http://www.cusdmathcoach.com/developing_fluency_with_basic_number_facts.pdf



Fact Mastery Con't

Strategy	Representative use to solve
Counting-on-from-the-first-addend	"2 . . . 3, 4, 5, 6" or "3, 4, 5, 6"
Counting-on-from-the-larger-addend	"4 . . . 5, 6" or "5, 6"
Linking	"2 + 2 = 4, + 2 more = 6"
Retrieval	"6"

Garnett, K. (1992). Developing fluency with basic number facts: Intervention for students with learning disabilities. Retrieved from http://www.cusdmathcoach.com/developing_fluency_with_basic_number_facts.pdf



Other Strategies

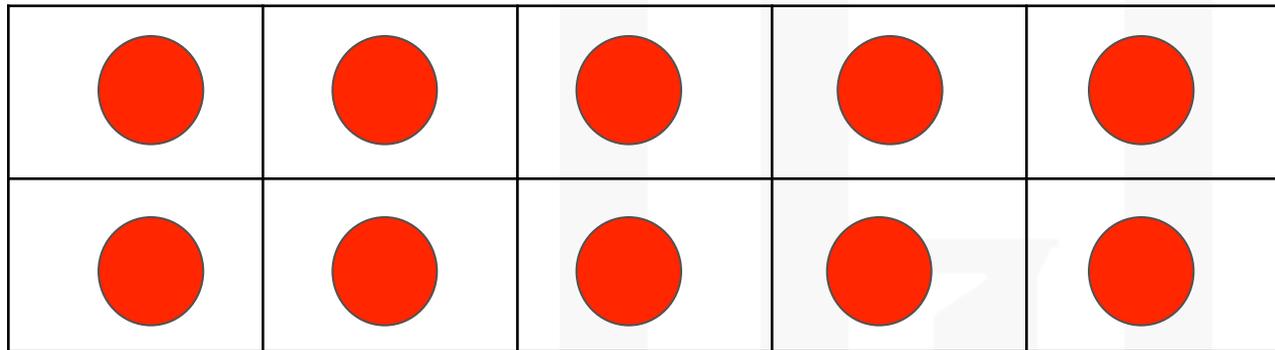
- One More, Two More





Make 10

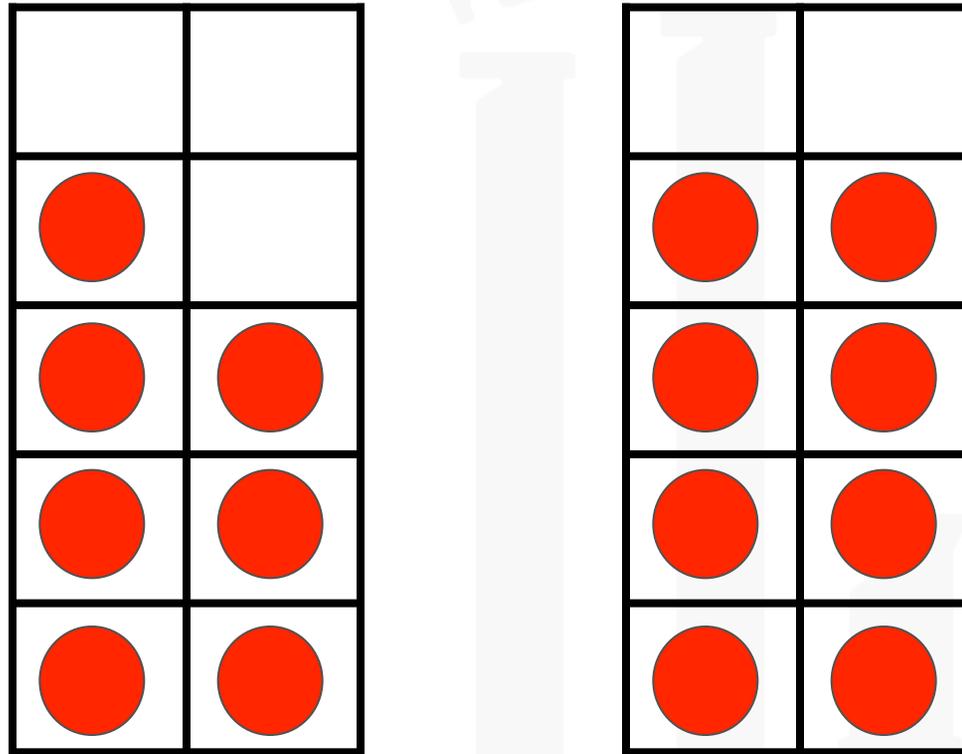
$$7 + \underline{\quad} = 10$$





Over 10

• $7 + 8 =$



$$7 + 8 = 10 + 5 = 15$$



Place Value and Addition

$$9 + 7 =$$

$$(9 + 1) + 6 =$$

$$10 + 6 = 16$$



Place Value and Addition

$$32 + 43 =$$

$$(30 + 2) + (40 + 3) =$$

$$(30 + 40) + (2 + 3) =$$

$$70 + 5 = 75$$



Regrouping and Addition

Step 1 Add the ones.
Regroup if needed
Example- 12 ones
becomes 1 ten and
2 ones

$$\begin{array}{r} 97 \\ + 35 \\ \hline 132 \end{array}$$

Step 3
Add the the
groups.
Example: 100
+30 +2=

Step 2 Add the
tens.
Regroup if needed
Example - 12 tens
becomes 1 hundred
2 tens



Instruction in Number Relationships

- Problem-solving contexts
- Use manipulatives or pictorial representations



Instruction in Number Relationships

- Students should communicate about what they have done and what they are thinking
- Help students recognize when to move to more efficient strategies (e.g., counting on, counting-on-from-the-larger-addend, making tens, etc.)

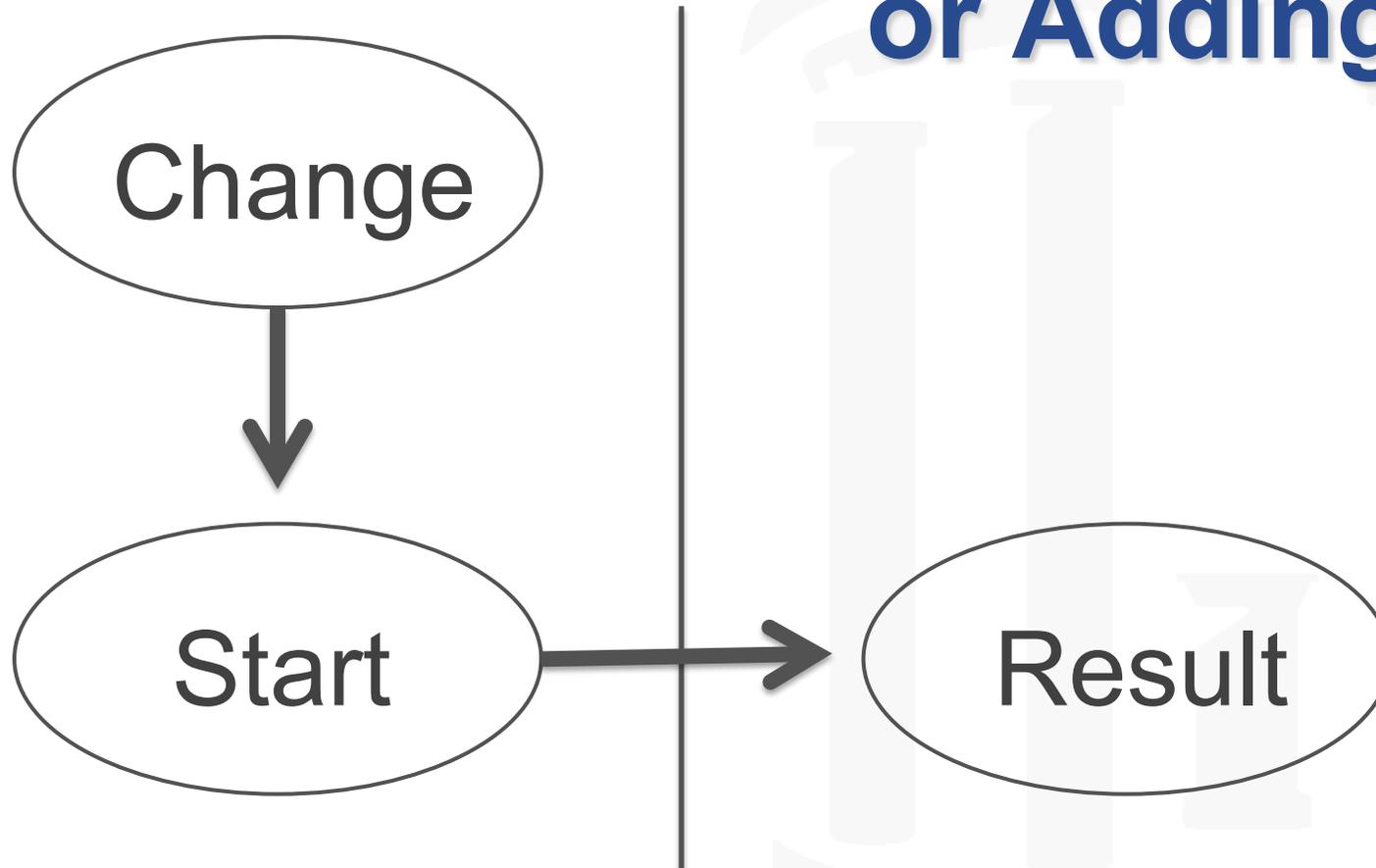


Activity 5

Strategies for Facts

1. Mark each strategy for learning to calculate accurately as recommended or not recommended.
2. For the strategies in the recommended box, discuss how students who use augmentative and alternative communication systems might demonstrate the strategy after they have provided an answer. The student might respond to a question like, “How do you know?”

Structures- Joining or Adding





Structures- Joining or Adding

- Result is Unknown: Lori had 6 pennies. Pam gave her 2 more. How many pennies does Lori have altogether?



Start

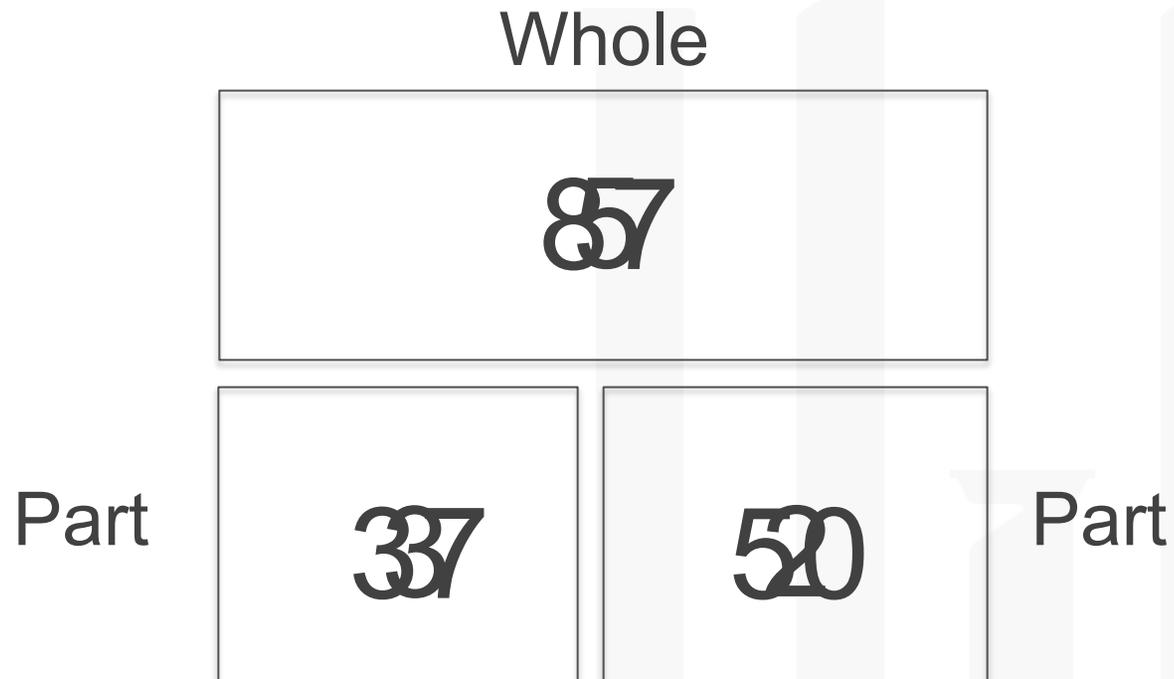


Result





Structures- Part-Part-Whole





Structures- Part-Part-Whole

- One Part Unknown: Dreama has 6 coins. 4 of her coins are pennies, and the rest are nickels. How many nickels does Dreama have?

Whole



Part

4



Part



Structures- Comparisons

Bigger Quantity

Smaller Quantity

Difference



Structures- Comparisons

- Smaller Bigger Unknown: Claire has 6 pennies. George has 4 more pennies than Claire. How many pennies does George have?

10

6

4



Activity 6

Problem Structures

1. Decide what structure type the story problem represents (join, part-part-whole, compare).
2. Rank order the problems in each structure type from easiest to hardest (1 being the easiest and 3 being the hardest)
3. Describe two sets of manipulatives that could be used to model each of the problem structures (join, part-part-whole, compare).
4. Describe two ways students could model each of the problem structures (join, part-part-whole, compare) without the use of their hands.



Properties of Addition

- Commutative Property:

$$a + b = b + a$$

- Associative Property:

$$(a + b) + c = a + (b + c)$$

- Identity Property

$$a + 0 = a$$



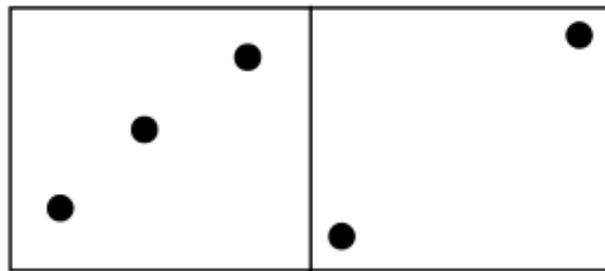
Commutative Property



$$3 + 5 = 5 + 3$$



Commutative Property

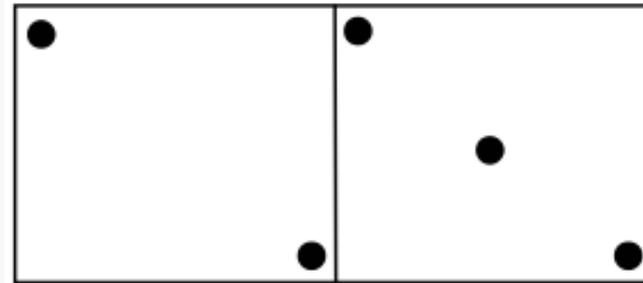


3

+

2

= 5



2

+

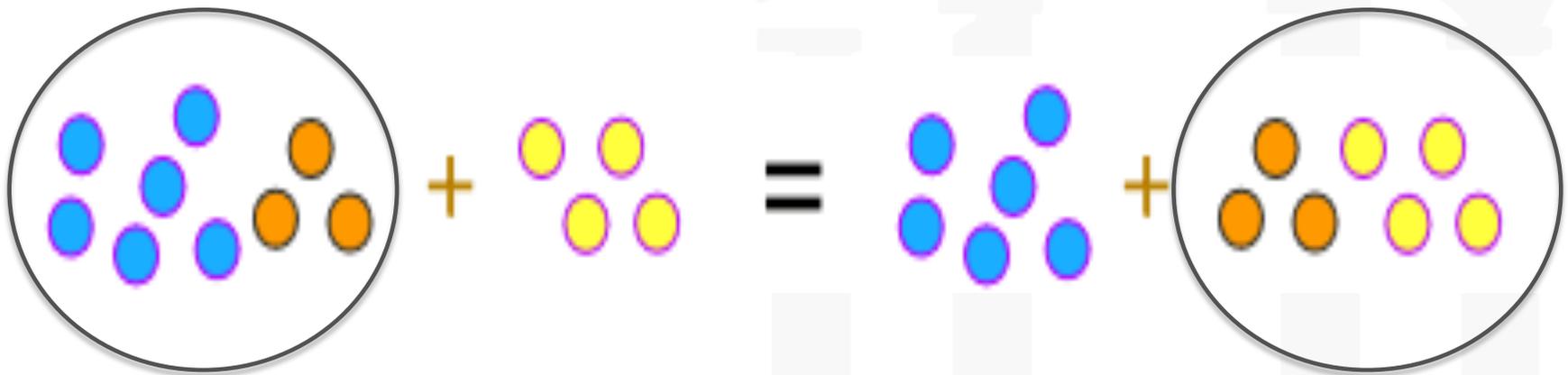
3

= 5

Does it change the total amount when we flip the domino?



Associative Property



$$(6 + 3) + 4$$
$$+ 4)$$

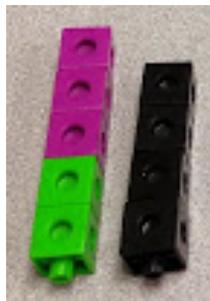
$$= 6 + (3$$
$$+ 4)$$



Associative Property



$$\begin{aligned} &(3 + 2) + 5 \\ &= 3 + (2 + 5) \end{aligned}$$





Identity Property

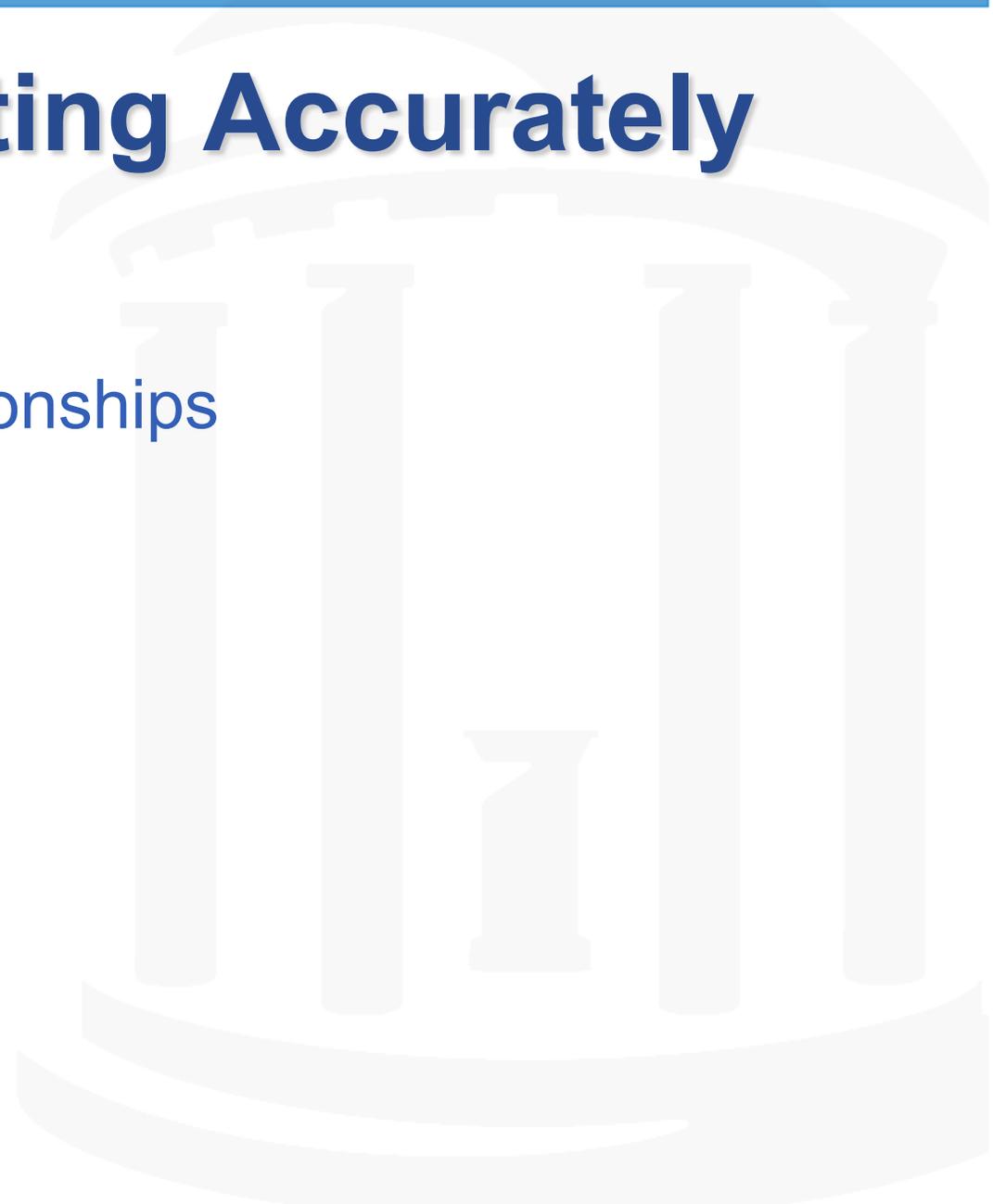


$$5 + 0 = 5$$



Calculating Accurately

- Number Relationships
- Structure
- Properties





Break



Algebraic Thinking



Instruction Aligned with DLM

- Think creatively and flexibly
- Use strategies that encourage students to use the known to solve unknown



Algebraic Thinking...

Elementary School

Understanding number relationships

Understanding equations: $3 + 2 = 5$;
 $9 - \square = 4$

And inequalities: $< > \neq :$ $4 < 5$



Algebraic Thinking...

Middle School

$$30 + x = 36$$

$$\text{Area} = L \times W$$

Blanton, M. L. (2008). Algebra and the elementary classroom: Transforming thinking, transforming practice. Portsmouth, NH: Heinemann.



Algebraic Thinking...

High School

$$\text{Volume} = L \times W \times H$$

$$32 \div x = 8$$

$$x = 4$$

Blanton, M. L. (2008). Algebra and the elementary classroom: Transforming thinking, transforming practice. Portsmouth, NH: Heinemann.



Approach to Algebraic Thinking

- Provide multiple means of representation
- Ask questions that encourage algebraic thinking

Blanton, M. L. (2008). Algebra and the elementary classroom: Transforming thinking, transforming practice. Portsmouth, NH: Heinemann.



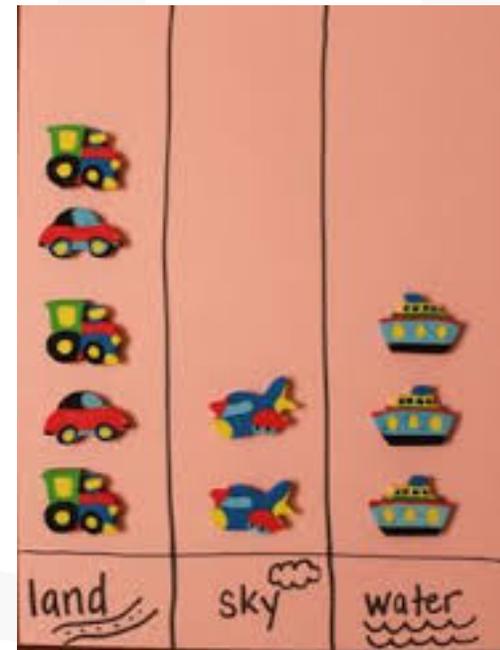
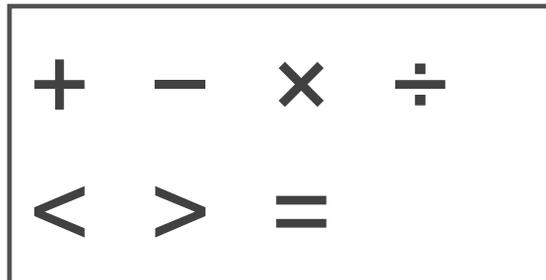
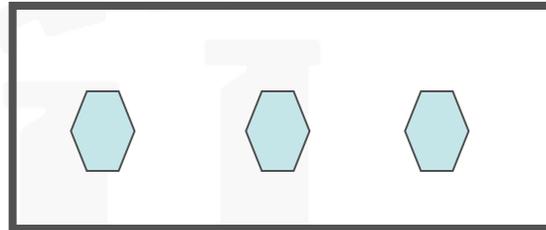
Approach to Algebraic Thinking

- Build on student understandings
- Help students make connections, describe, and justify general patterns and regularities in operations and properties of number

Blanton, M. L. (2008). Algebra and the elementary classroom: Transforming thinking, transforming practice. Portsmouth, NH: Heinemann.



Representation



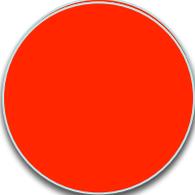
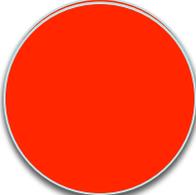
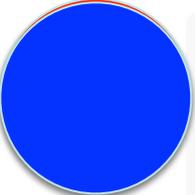
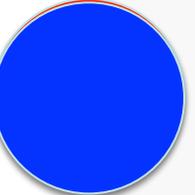
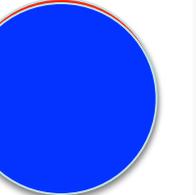
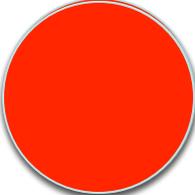
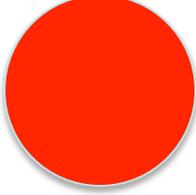


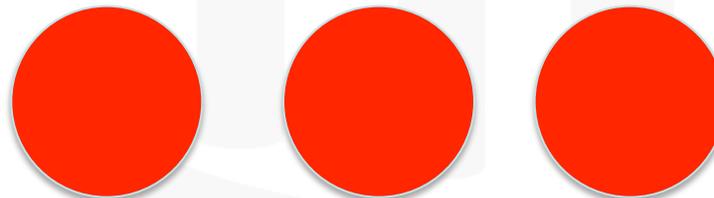
Use Representations to...

- organize, record, and communicate.
- select, apply, and translate to solve problems.
- model and interpret mathematical equations.



Observe Student Thinking





Activity 7

What is Algebraic Thinking?

1. Each statement refers to either algebraic thinking or arithmetic calculation.
2. Mark each of the statements with A for algebraic thinking or a C for calculation.



The Equal Sign

$$7 + 4 = \square$$

$$7 + 4 = \square + 5$$

= means “the same quantity as”



Equality and Number Sentences

Equality is the relationship expressed by the idea that two mathematical expressions hold the same value.

$$5 + 7 = 6 + 6$$

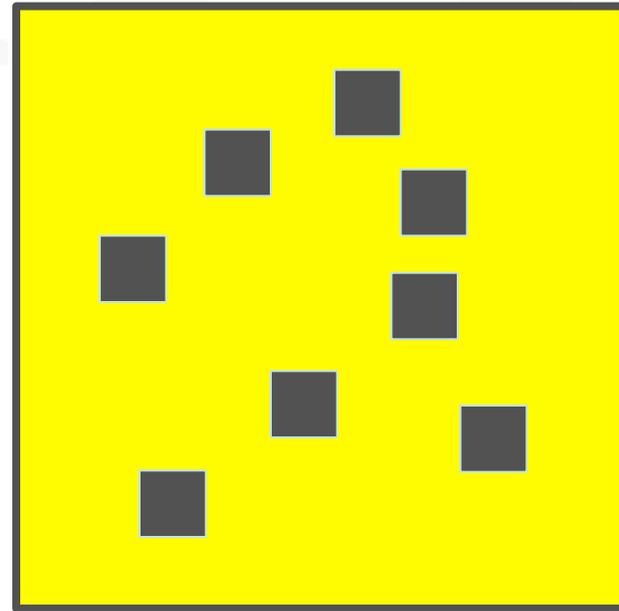
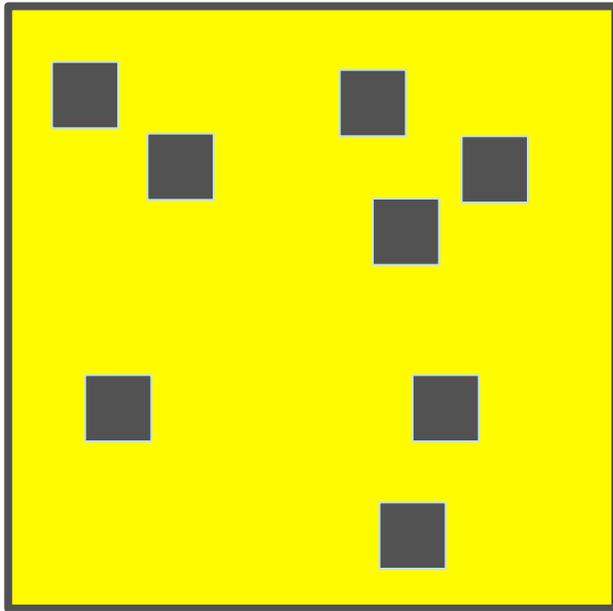
True

$$4 + 3 = 5 + 5$$

False



Comparing Numbers



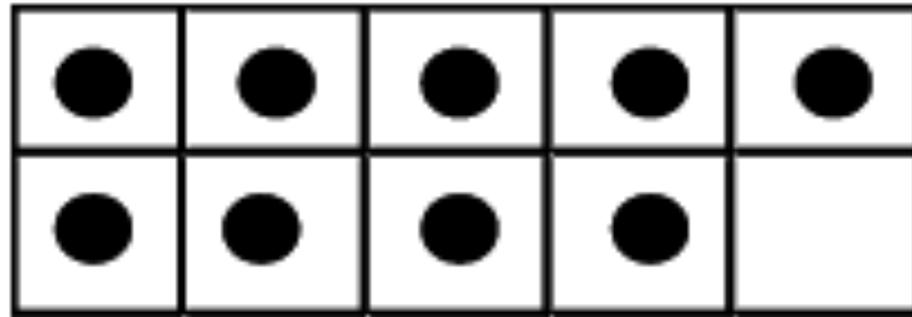
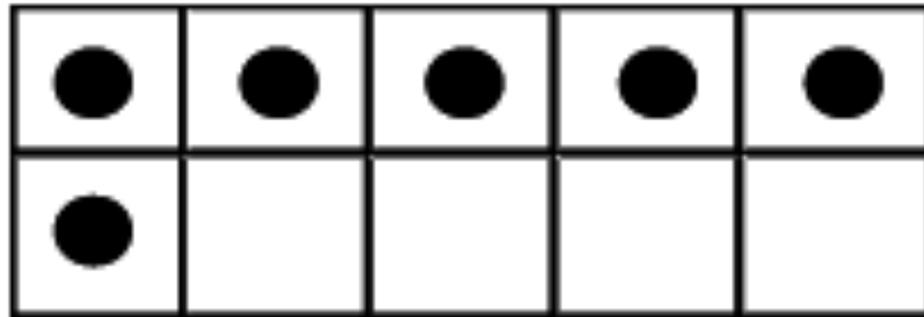
Same

$$8 = 8$$



Comparing Quantity

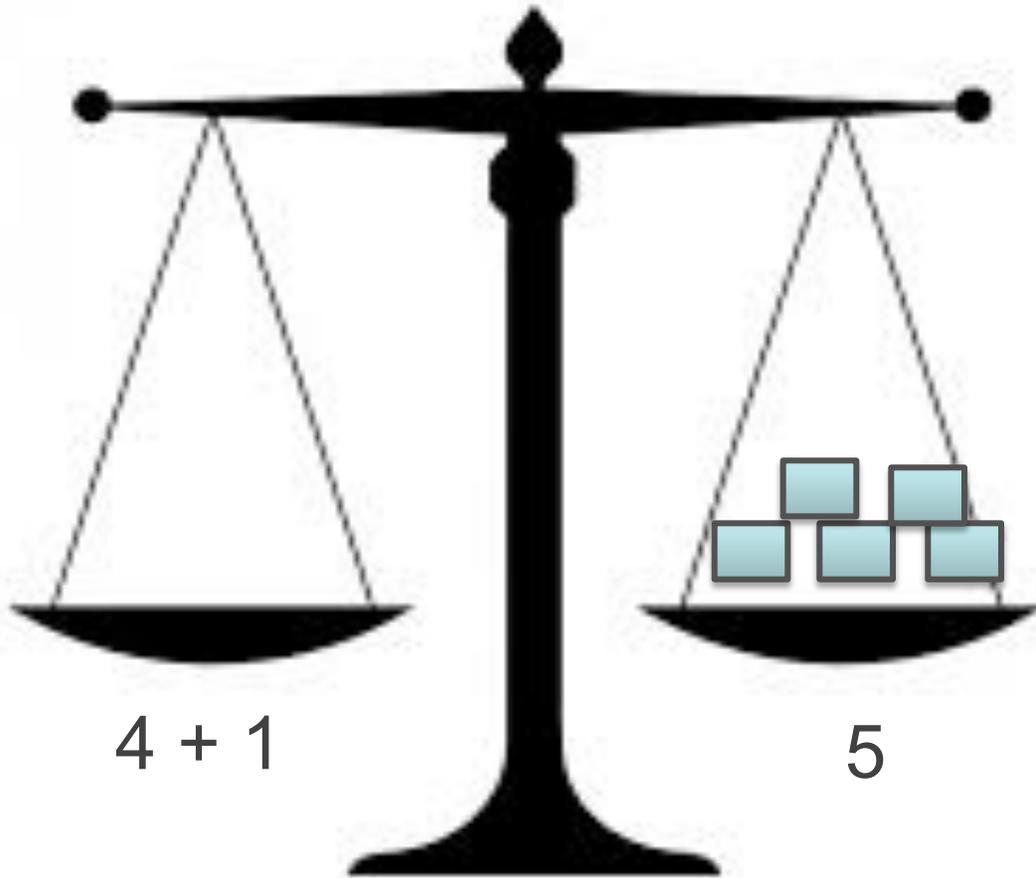
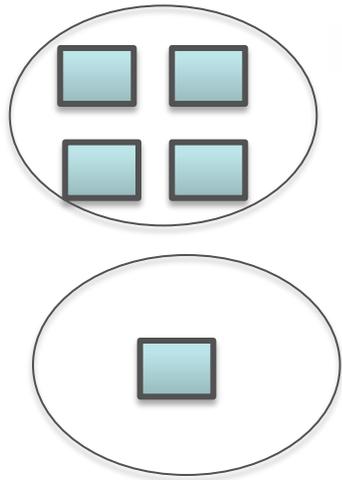
Comparing
ten-frames





The Same Quantity As

$$4 + 1 = 5$$





$$5 + x = 9$$





Understanding Equality

$$3x + 20 = 47$$

$$- 20 \quad - 20$$

$$3x = 27$$

$$3 \times \square = 27$$

$$3 \times 9 = 27$$

$$x = 9$$

Falkner, K. P., L. Levi, and T. P. Carpenter (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics*, 6(1), p. 234.



Generalizations

$$9 + 4 = 13 \text{ (odd)}$$

$$11 + 2 = 13 \text{ (odd)}$$

$$7 + 6 = 13 \text{ (odd)}$$

$$8 + 5 = 13 \text{ (odd)}$$

$$2 + 3 = 5 \text{ (odd)}$$

$$19 + 10 = 29 \text{ (odd)}$$

$$23 + 46 = 69 \text{ (odd)}$$



Activity 8

What Does Equal Mean?

1. Find the unknown number that balances the scale.
2. Discuss how the scale could be used to teach equality to students who are just beginning to develop number sense and early operations.

Generalizations

$$78 + 0 = 78$$

$$13 + 0 = 13$$

$$10 + \underline{\quad} = 10$$

When you add zero to a number, you get the number you started with.

$$96 - 96 = 0$$

$$22 - 22 = 0$$

$$5 - \underline{\quad} = 0$$

When you subtract a number from itself, you get zero.



$$96 \times 0 = 0$$

$$17 \times 0 = 0$$

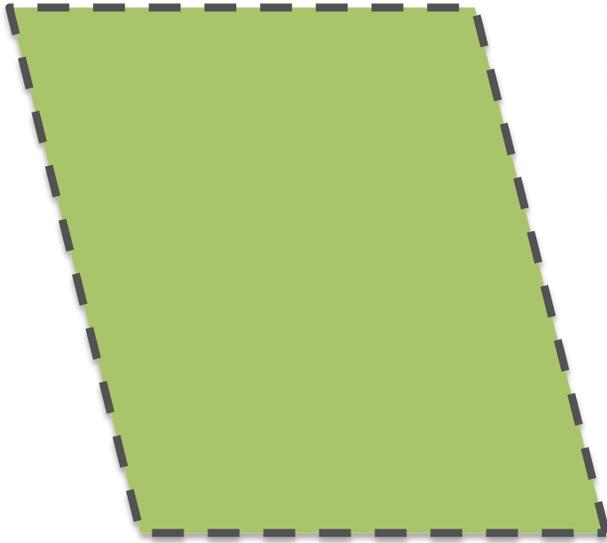
$$2 \times \underline{\quad} = 0$$

When you multiply a number times zero, you get zero.

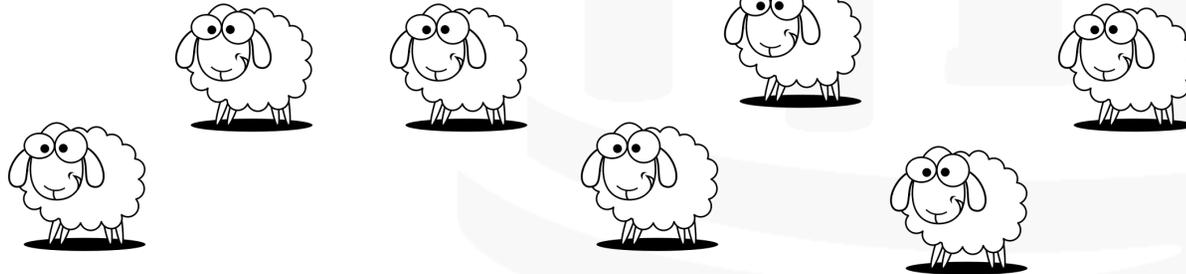
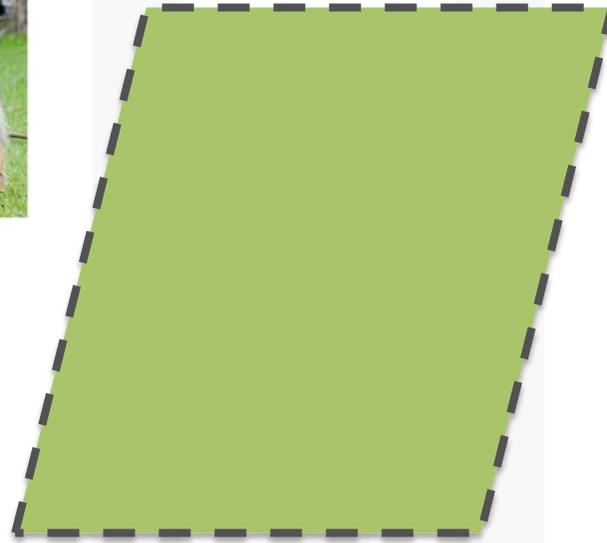


Generalizations

$$3 + 0 = 7$$



$$6 + 1 = 7$$





$$n + 1 = \frac{\quad}{\quad}$$





- Provide multiple means of representation
- Ask questions that encourage thinking
- Build on student understandings
- Help students make connections, describe, and justify general patterns and regularities in operations and properties of number



Wrap-up